**To Prove :**

GK $∥$ BC

It is enough to prove $\frac{FG}{GB}=\frac{FK}{KC} $(from converse of BPT)

**Proof:**

In $∆ FBC; $cevians FD, BA, CG are concurrent at E.

$∴$ $\frac{FG}{GB}×\frac{BD}{DC}×\frac{CA}{AF}=1$

$$⇒\frac{FG}{GB}×\frac{1}{2}×\frac{CA}{AF}=1$$

$⇒\frac{FG}{GB}=\frac{2AF}{AC}$ -----------------------------(1)

In $∆ABC,$ ceivans AD, BK, CF are concurrent at H

$∴$ $\frac{AE}{EB}×\frac{BD}{DC}×\frac{CK}{KA}=1$ [from Ceva's theorem]

$⇒\frac{AE}{EB}×\frac{1}{2}×\frac{CK}{KA}=1$

$⇒\frac{AE}{EB}=\frac{2AK}{CK}$ -------------------------- (2)

In $∆ABC,$ DEF is transversal

$∴$ $\frac{BD}{DC}×\frac{CF}{FA}×\frac{AE}{BE}=1$ (from 'Menaulas Theorem')

$$⇒\frac{1}{2}×\frac{CF}{CA}×\frac{AE}{BE}=1$$

$\frac{AE}{BE}×\frac{2AF}{CF}$ --------------------- (3)

from (2) & (3)

$$\frac{AK}{CK}×\frac{AF}{CF}$$

Adding '1' on both sides

$$⇒\frac{AK}{CK}+1=\frac{AF}{CF}+1$$

$$⇒\frac{AK+CK}{CK}=\frac{AF+CF}{CF}$$

$$⇒\frac{AC}{CK}=\frac{AF+CF}{CF}$$

$$⇒\frac{CF}{CK}=\frac{AF+CF}{AC}$$

subtracting '1' on both sides

$$\frac{CF}{CK}-1=\frac{AF+CF}{AC}-1$$

$\frac{CF-CK}{CK}=\frac{AF+CF-AC}{AC}$

$\frac{FK}{KC}=$ $\frac{AF+CA+AF-AC}{AC}$ [$∵CF=CA+AF]$

$\frac{FK}{KC}=\frac{2AF}{AC}$ ------------------------------(4)

from (1) & (4)

$\frac{FG}{GB}=\frac{FK}{KC}$

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Which implies GK $∥ $BC from converse of basic proportionality theorem.

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